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A MODEL FOR ESTIMATING RADIOWAVE ACQUISITION RECEIVING SYSTEM INTERCEPT PROBABILITY

Jeffrey B. Knorr

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This report describes a method for estimating radiowave acquisition receiving system intercept probability. Performance data for two or more receiving systems is required to implement the method.

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SUMMARY

This report presents a model for estimating the intercept probability of a radiowave acquisition receiving system. The specific configuration or type of receiving system is not important but there must be two or more receiving systems and they must operate in such a manner that for each trial (event) they alarm independently from a statistical point of view. The data giving the number of alarms for each receiver and the number of receivers alarming for each trial is all that is required. These data are used to form estimates of the conditional probabilities of intercept and simultaneous alarm. The estimates are then used in a least mean square calculation from which the unconditional probabilities are estimated.

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I. INTRODUCTION

A. Background

In the conduct of electronic warfare it is necessary to employ radio receivers to monitor activity in the electromagnetic spectrum for a variety of purposes. In particular, electronic support measures (ESM) and signal intelligence (SIGINT) gathering are concerned with this monitoring process. The acquisition of an electromagnetic signal by a radio receiver is referred to as intercept.

Two important considerations with regard to signal intercept are the probability of intercept and the mean time to intercept. In general, one has no a priori knowledge that a signal will be present at a given time. Further, if a signal is present, the frequency and location of the emitter may not be known. Thus it is most generally necessary to search in time, frequency and space in order to intercept a signal. The probability that the signal will be intercepted at all is of interest. Additionally, if a signal is intercepted it is of interest to know the mean time to intercept. This is particularly important in cases where the signal represents a threat since if it is not intercepted in a timely manner the results may be catastrophic.

There are currently five major types of receivers being used for the acquisition of signals. These are:

(1) Crystal video

(2) IFM

(3) Sweeping superheterodyne

(4) Compressive

(5) Channelized (including Bragg)

These receivers are sometimes used in combinations and in addition, circuitry for the automatic recognition of signals may be employed at the receiver output. An acquisition receiving system may thus be quite complex. The determination of the intercept probability of such an acquisition system is correspondingly complex.

B. Related Work

The subject of intercept probability has certainly been of interest for as long as radio receivers have been used to search for signals. Early literature on this subject may be found in ref [1] where sweeping superheterodyne receivers are discussed. This work shows that receiver intercept performance generally improves as the sweep rate is increased.

As technology progressed, other types of receivers were developed and used in complex system configurations as discussed above. A reasonably comprehensive search of the technical literature indicates, however, that little has been done to advance our understanding of the intercept probability problem to the point where this can be quantified for an acquisition system.

C. Problem

This report addresses the problem of estimating radiowave acquisition receiving system intercept probability. A method for estimating this probability from system performance data is presented. An attractive feature of the method is the fact that the particular system configuration

does not enter directly into the calculations. Certain assumptions must be satisfied, however, and the actual hardware configuration may enter into the problem indirectly in this way.

II. Probabalistic Intercept Theory

A. Two Acquisition Receiving Systems

With reference to Figure 1, assume that two acquisition receiving systems are excited by a common source emitter. The emission will generally be of finite duration and will occur at unknown time, frequency and location. The receivers may be collocated and tied to a common antenna in which case they share a common channel or they may be physically separated in which case the channels may have quite different characteristics. Further, assume that the intercept probabilities for the two receiving systems (including the effects of the channel) are p_{I1} and p_{I2} and that the systems behave independently in the strict statistical sense. For any emission (experiment) there can be one of four outcomes. Only system 1 may alarm, only system 2 may alarm, both may alarm or neither may alarm. Let $P(n)$ be defined as follows:

$P(n)$ = probability of n simultaneous alarms

where

$$n \in \{0,1,2\}.$$

It is easy to show that

$$\begin{aligned} P(0) &= (1-p_{I1})(1-p_{I2}) \\ P(1) &= p_{I1}(1-p_{I2}) + p_{I2}(1-p_{I1}) \\ P(2) &= p_{I1}p_{I2} \end{aligned} \tag{1}$$

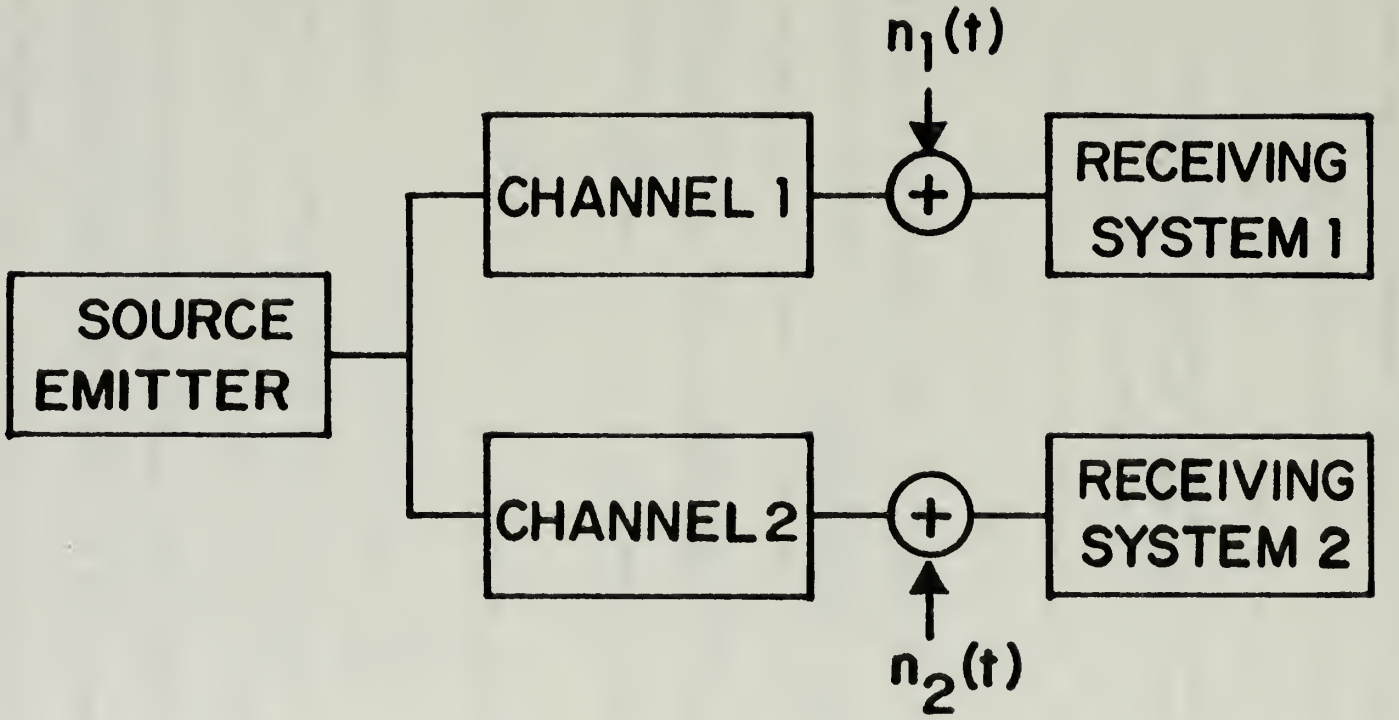

Figure 1. Model for radiowave signal acquisition by two receiving systems.

Consider the following examples:

- -

Example 1. Calculate the (simultaneous) alarm probabilities for two acquisition receiving systems with $p_{I1} = 0.8$ and $p_{I2} = 0.7$.

Solution: It is easy to show using equation (1) that

$P(0) = .06$

$P(1) = .38$

$P(2) = .56$

In this case there is a 94% probability that either one or both systems will alarm and the signal will be intercepted.

Example 2. Calculate the alarm probabilities for two acquisition receiving systems with $p_{I1} = 0.3$ and $p_{I2} = 0.4$.

Solution: Again using equation (1)

$P(0) = .42$

$P(1) = .46$

$P(2) = .12$

In this case there is only a 58% probability that either one or both systems will alarm and the signal will be intercepted. Note also that the most probable outcome is for only one alarm whereas in the previous example the most probable outcome was for two alarms.

- -

At this point it seems appropriate to comment further on the independence of the receiving systems (including channel). This is a point which is naturally of considerable interest. The assumption of total independence represents one extreme; the other would be total dependence in which case both receiving systems would behave identically. The interesting question, of course, is where in this interval the dependence lies in any particular case. This is a question which probably cannot be answered in general. The variety of possible acquisition receiving system configurations and applications is such that an exhaustive examination of the many possibilities would be impossible. There appear to be situations where this assumption is reasonable but no claim of applicability is made for any particular case. Each reader must decide if the model described here will fit his own case. The assumption of independence has one important implication from a mathematical point of view. It makes the problem tractable using the method described here.

B. N Acquisition Receiving Systems

We now generalize the previous theory to the case of N acquisition receiving systems with intercept probabilities $p_{I1}, p_{I2}, \ldots, p_{IN}$. It is again assumed that the systems are statistically independent. For this case, it is shown in Appendix I that the alarm probabilities $P(n)$ may be calculated using the algorithm shown in Table I.

Step	Computation
1	Calculate $x_j = p_{Ij}/(1-p_{Ij})$
2	Expand $\prod_{j=1}^{N} (x-x_j)$ to obtain the polynomial $\prod_{j=1}^{N} (x-x_j) = a_0 x^N + a_1 x^{N-1} + a_2 x^{N-2} + \ldots + a_N$
3	Calculate $P(n) = \dfrac{\lvert a_n \rvert}{\sum_{k=0}^{N} \lvert a_k \rvert}$

Table 1. Algorithm for Computation of Alarm Probabilities P(n).

- -

Example 3. Use the algorithm above to calculate P(n) for the intercept probabilities of example 1.

Solution: Using the algorithm above,

Step 1. $x_1 = 4$

$x_2 = 7/3$

Step 2. $\prod_{j=1}^{2} (x-x_j) = (x-4)\ (x-7/3)$

$= x^2 - \frac{19}{3} x + \frac{28}{3}$

Step 3. $\sum_{k=0}^{2} |a_k| = 1 + \frac{19}{3} + \frac{28}{3} = \frac{50}{3}$

$P(0) = \frac{1}{50/3} = .06$

$P(1) = \frac{19/3}{50/3} = .38$

$P(2) = \frac{28/3}{50/3} = .56$

It can be seen that the results agree with those obtained in example 1.

- -

The algorithm is simple although the computational labor grows rapidly as N increases. Practically, however, the important result is that a simple algorithm exists. The computational labor can be delegated to a computing machine.

C. Discussion of the Probability Law for Alarms

The mean and variance of the probability law are most easily obtained in the following way. Define random variables

$$X_j = \begin{cases} 1 & \text{if acquisition system } j \text{ alarms} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and then for each trial number of alarms is

$$n = \sum_{j=1}^{N} X_j \tag{3}$$

It then follows since the X_j are independent that

$$E[n] = \sum_{j=1}^{N} p_{Ij} \tag{4}$$

$$E\left[(n-\bar{n})^2\right] = \sum_{j=1}^{N} p_{Ij}\,(1-p_{Ij}) \tag{5}$$

- -

Example 4. Calculate the mean and variance for alarms using the p_{Ij} of example 1.

Solution: Using Eq (4) and Eq (5),

$E[n] = 1.5$

$E\left[(n-\bar{n})^2\right] = .37$

This result is intuitively satisfying since example 1 showed the most probable outcomes to be either 1 or 2 alarms.

- -

An interesting case results when all the receiving systems have the same intercept probability $p_{Ij} = p_I \; \forall j$. In this case the probability law reduces to the Binomial law

$$P(n) = \binom{N}{n} p_I^N \; (1-p_I)^{N-n} \tag{6}$$

$$n \in \{0,1,2,\ldots,N\}$$

which has mean

$$E[n] = N \, p_I \tag{7}$$

and variance

$$E\left[(n-\bar{n})^2\right] = N \, p_I \, (1-p_I) \tag{8}$$

Lengthy discussions of this probability law can be found in any probability text.

- -

Example 5. Calculate the probability law, mean and variance for alarms when $N=5$ and $p_{Ij} = .5 \; \forall j$.

Solution: The Binomial law Eq (6) - (8) applies and we have

$$P(0) = 1 \; p_I^0 \; (1-p_I)^5 = 1/32$$

$$P(1) = 5 \; p_I^1 \; (1-p_I)^4 = 5/32$$

$$P(2) = 10 \; p_I^2 \; (1-p_I)^3 = 10/32$$

$$P(3) = 10 \; p_I^3 \; (1-p_I)^2 = 10/32$$

$$P(4) = 5 \; p_I^4 \; (1-p_I)^1 = 5/32$$

$$P(5) = 1 \; p_I^5 \; (1-p_I)^0 = 1/32$$

with

$$E[n] = N P_I = 2.5$$

and

$$E[(n-\bar{n})^2] = N P_I (1-P_I) = 1.25$$

This shows that there is approximately a 60% probability of either 2 or 3 alarms for each trial and that there will be little deviation (40%) from that result.

III. Estimation of Intercept Probabilities

In most cases it would be difficult to calculate the intercept probability of a receiving system. If system performance data are available, however, it may be possible to estimate intercept probability. This section presents a method for estimating the intercept probabilities of two or more receiving systems if the systems are statistically independent.

A. Two Acquisition Receiving Systems

Let us make the following definitions:

A_1 = event: receiver 1 alarms

A_2 = event: receiver 2 alarms.

It then follows that (see Figure 2)

$$P(A_1) = P(A_1 \mid A_1 \cup A_2)\ P\ (A_1 \cup A_2) \tag{9}$$

$$P(A_2) = P(A_2 \mid A_1 \cup A_2)\ P\ (A_1 \cup A_2) \tag{10}$$

$$P(A_1A_2) = P\ (A_1A_2 \mid A_1 \cup A_2)\ P\ (A_1 \cup A_2)\ . \tag{11}$$

Dividing Eq (11) by Eq (9) and then Eq (10) we obtain

$$\frac{P(A_1A_2)}{P(A_1)} = \frac{P(A_1A_2 \mid A_1 \cup A_2)}{P(A_1 \mid A_1 \cup A_2)} \tag{12}$$

and

$$\frac{P(A_1A_2)}{P(A_2)} = \frac{P(A_1A_2 \mid A_1 \cup A_2)}{P(A_2 \mid A_1 \cup A_2)}\ . \tag{13}$$

Now if we assume independence

$$P(A_1A_2) = P(A_1)\ P(A_2) \tag{14}$$

and we substitute Eq (14) into Eqs (12) and (13) we obtain

$$P(A_1) = \frac{P(A_1\ A_2 \mid A_1 \cup A_2)}{P(A_2 \mid A_1 \cup A_2)} \tag{15}$$

$$P(A_2) = \frac{P(A_1 A_2 \mid A_1 \cup A_2)}{P(A_1 \mid A_1 \cup A_2)} \quad . \tag{16}$$

Using a prime to denote conditional probability we may write Eqs (15) and (16) more compactly as

$$p_{I1} = \frac{p'_{I12}}{p'_{I2}} \tag{15a}$$

$$p_{I2} = \frac{p'_{I12}}{p'_{I1}} \tag{16a}$$

If receiving system data are available as shown in the Venn diagram of Figure 2, then the intercept probabilities may be estimated on the basis of relative frequency as

$$\hat{p}_{I1} = \frac{n_{12}}{n_{12} + n_2} \tag{17}$$

$$\hat{p}_{I2} = \frac{n_{12}}{n_{12} + n_1} \tag{18}$$

where

n_1 = # times only receiver 1 alarms

n_2 = # times only receiver 2 alarms

n_{12} = # times both receivers alarm.

- -

Example 6. Given the data below, estimate p_{I1}, p_{I2} and signals missed.

Solution: Using Eqs (17) and (18) with

$n_1 = 30$

$n_2 = 40$

$n_{12} = 50$

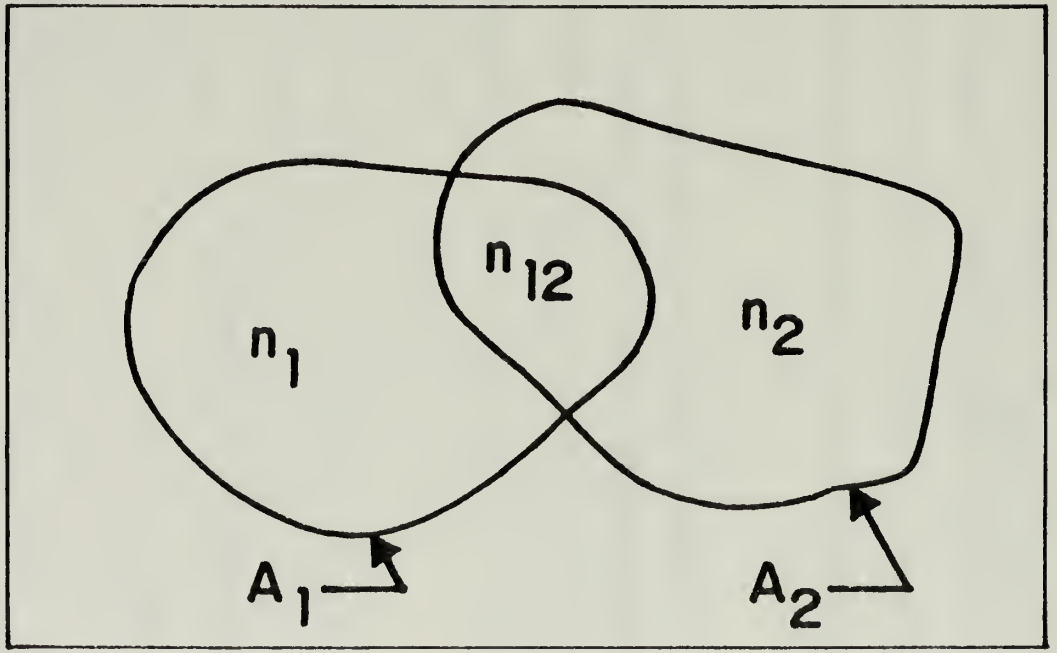

Figure 2. Venn diagram showing number of occurences for the various outcomes when two receiving systems are used to attempt intercept of a radio signal.

we obtain

$$\hat{p}_{I1} = \frac{50}{50 + 40} = 5/9$$

$$\hat{p}_{I2} = \frac{50}{50 + 30} = 5/8$$

We may now estimate the probability that no alarm occurred as

$$\hat{P}(\bar{A}) = (1-\hat{p}_{I1})\ (1-\hat{p}_{I2}) = 1/6$$

and it follows that

$$\hat{P}(A) = 5/6 \ .$$

Defining

n_0 = # trials for which no alarm occurred

n_T = total trials

we have

$$n_T\ P(A) = n_1 + n_2 + n_{12}$$

or

$$\hat{n}_T = \frac{6}{5} \times 120 = 144 \ .$$

The estimate of signals missed is therefore

$$\hat{n}_0 = \hat{P}(\bar{A})\ \hat{n}_T = 24 \ .$$

- -

The above example illustrates a very interesting result. If we have two independent receivers we can not only estimate their intercept probabilities but also the probability that a signal will be missed as well as the number of such signals.

B. N Acquisition Receiving Systems

We will now investigate the extension of the method just presented to N receiving systems. In this case if we attempt to simply extend the method just described for N = 2 to the case N = 3 we find that there are a number of possible ways to estimate the p_{Ij}. It is not clear which way this should be done except perhaps on the basis of intuition regarding the relative amounts of data. It may be expected that this dilema would become more difficult as N increases. Thus, in an attempt to circumvent this difficulty and to make maximum use of the available data a least squares approach was selected.

Consider the data obtained if N receiving systems are simultaneously employed. From this data we can estimate the following quantities on the basis of relative frequency

$$\hat{P}'_{I1}, \hat{P}'_{I2}, \hat{P}'_{I3} \cdots \hat{P}'_{IN}$$

$$\hat{P}'(1), \hat{P}'(2), \hat{P}'(3) \ldots \hat{P}'(N).$$

If our estimate that an alarm occurs at all is $\hat{P}(A)$ then

$$\hat{P}_{Ij} = \hat{P}'_{Ij}\,\hat{P}(A) \tag{19}$$

$$\hat{P}(0) = 1-\hat{P}(A)$$

$$\hat{P}(n) = \hat{P}'(n)\,\hat{P}(A) \qquad (n < 1) \tag{20}$$

Suppose that we now use the $\hat{P}_{Ij}$ in the alogrithm of Table 1 to estimate P(n). Denote this estimate as $\hat{\hat{P}}(n)$ and define the mean square weighted error as

$$E^2 = \sum_{n=0}^{N} W_n \left[\hat{P}(n) - \hat{\hat{P}}(n)\right]^2 \left[\frac{1}{N+1}\right] \tag{21}$$

For fixed weights W_j, the mean square error will vary with $P(A)$ as this quantity varies through the range

$$0 \leq P(A) \leq 1 \tag{22}$$

We simply choose as our estimate, $\hat{P}(A)$, that non-trivial value which minimizes the mean square error as given by Eq (21). The estimates for the p_{Ij} then follow from Eq (19).

The weights, W_n, in Eq (21) should be chosen so as to weight the data according to its reliability. One way to accomplish this is to use the inverse of the variance. A simple approach, however, is to set all weights equal to 1 ($W_n = 1 \; \forall n$) which automatically weights data associated with less probable outcomes less heavily.

A computer program has been written to carry out the computations just described for $N \leq 9$. $W_n = 1 \; \forall n$ has been used. The use of this program is illustrated in the following example.

- -

Example 7. Solve Example 6 using the method of least squares.

Solution: From the data given in Example 6 we calculate the following estimates for conditional probabilities:

$p'_{I1} = 80/120$ $\qquad$ $\hat{P}'(1) = 70/120$

$p'_{I2} = 90/120$ $\qquad$ $\hat{P}'(2) = 50/120$

If we now compute the mean square error as given by Eq (21) with $W_n = 1 \; \forall n$ and plot this as a function of $P(0) = [1-P(A)]$ we obtain the result shown in Figure 3. $P(0) = 1$ is a trivial case for which the error will always be $E^2(0) = 0$. A unique non-trivial minimum occurs for $\hat{P}(0) = 1/6 = .167$ where again $E^2(1/6) = 0$. This is exactly the answer obtained in Example 6.

The intercept probabilities may now be estimated as

$$\hat{p}_{I1} = \hat{p}'_{I1} \quad 1-\hat{P}(0) = 5/9$$

$$\hat{p}_{I2} = \hat{p}'_{I2} \quad 1-\hat{P}(0) = 3/8$$

which again is exactly the result obtained in Example 6.

- -

In Example 7 the method of least squares gives exactly the answer obtained by the simpler method employed in Example 6. It is generally true that

$$\min_{P(0)} E^2(P(0)) = 0 \qquad (N=2).$$

If $N > 2$ then the error is not necessarily zero at the non-trivial minimum. This is illustrated in the next example.

- -

Example 8. Estimate $P(0)$ and the p_{Ij} for the case $N = 5$ given the following data:

$\hat{p}'_{I1} = .81$ $\qquad$ $\hat{P}'(1) = .04$

$\hat{p}'_{I2} = .69$ $\qquad$ $\hat{P}'(2) = .25$

$\hat{p}'_{I3} = .60$ $\qquad$ $\hat{P}'(3) = .36$

$\hat{p}'_{I4} = .54$ $\qquad$ $\hat{P}'(4) = .22$

$\hat{p}'_{I5} = .36$ $\qquad$ $\hat{P}'(5) = .03$

Solution: As in Example 7, the error is calculated from Eq (21). This is plotted as a function of $P(0)$ in Figure 4. The non-trivial minimum occurs at $P(0) = .015$. The corresponding intercept probability estimates are

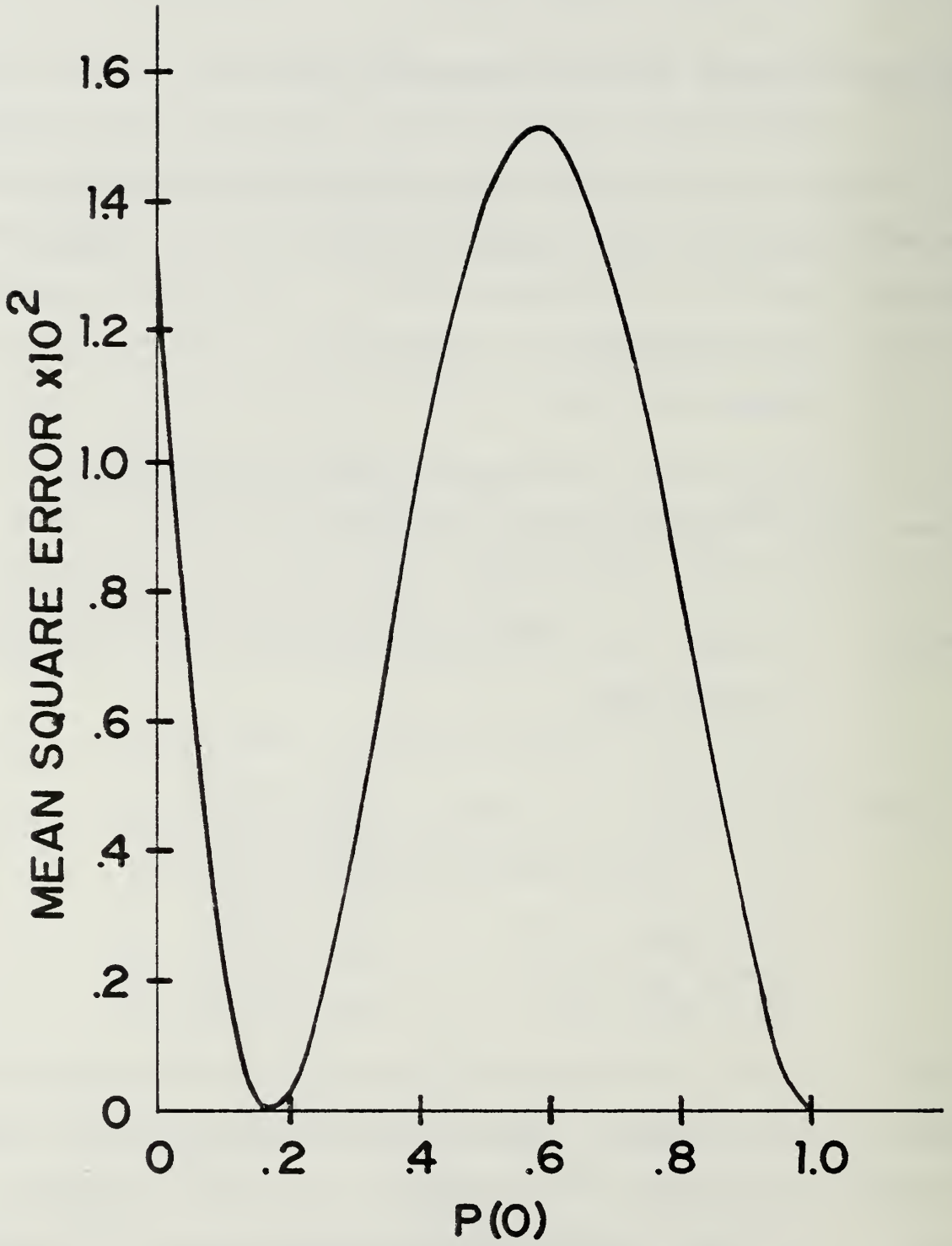

Figure 3. Mean square error vs. P(0) for data of Example 7 (N = 2).

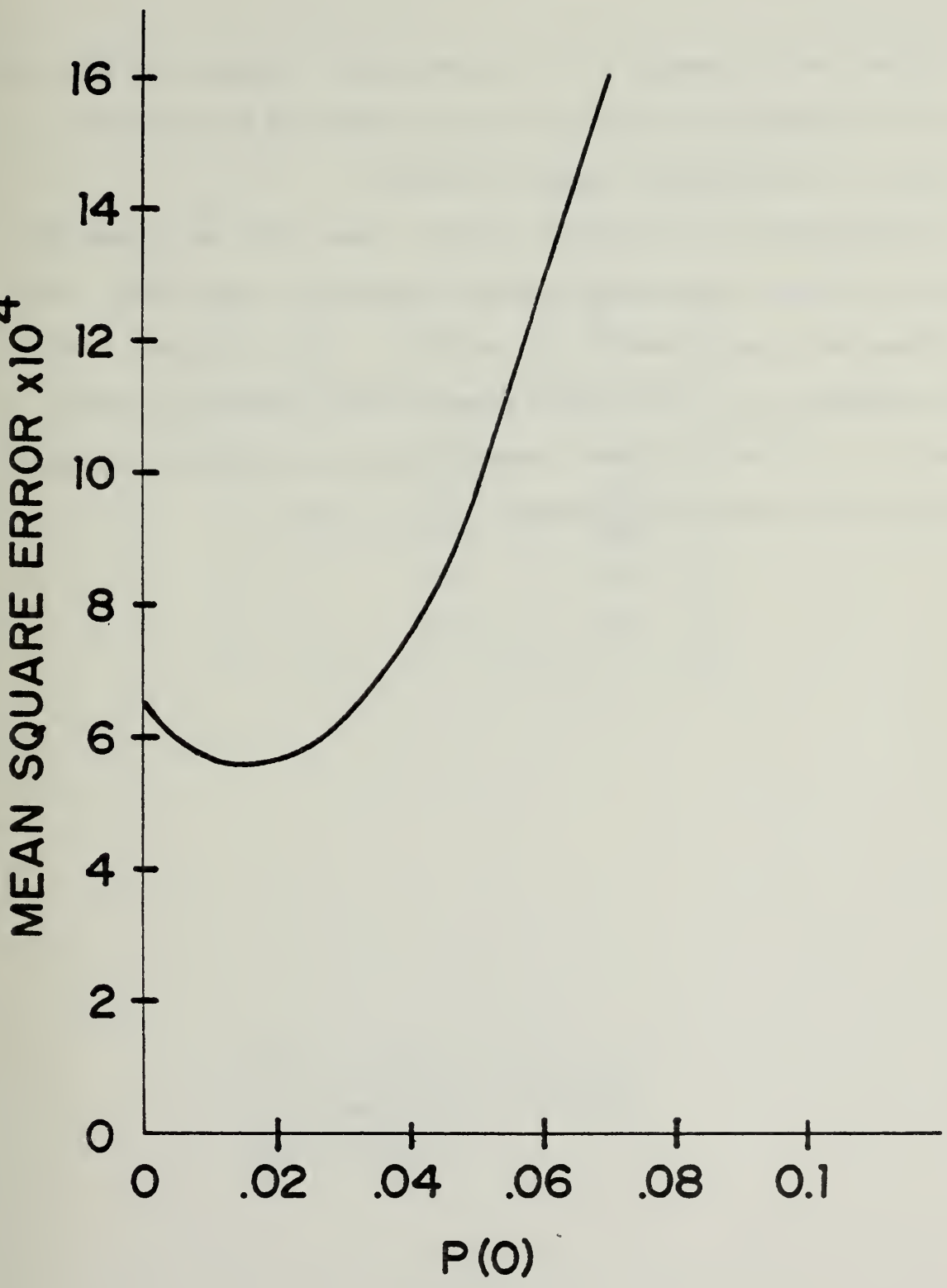

Figure 4. Mean square error vs. P(0) for data of Example 8 (N = 5).

$\hat{p}_{I1} = .797$ $\hat{p}_{I3} = .591$ $\hat{p}_{I5} = .354$

$\hat{p}_{I2} = .679$ $\hat{p}_{I4} = .531$

- -

For the data of Example 8 it is interesting to compare the $\hat{P}(n)$ and $\hat{\hat{P}}(n)$ at the minimum error point to see the effects of the weighting $W_n = 1 \forall n$. This comparison appears in Table 2.

An examination of the figures in Table 2 shows that the values for $2 \leq n \leq 4$ are very close while for other values of n the $\hat{P}(n)$ and $\hat{\hat{P}}(n)$ disagree by approximately a factor of 2. This is a direct result of the weighting $W_n = 1 \forall n$ which causes greater attention to those values of n for which a greater amount of data is available. In this case about 70% of the data corresponds to $2 \leq n \leq 4$.

n	$\hat{P}(n)$	$\hat{\hat{P}}(n)$
0	.0150	.0080
1	.0394	.0736
2	.2462	.2435
3	.3546	.3655
4	.2167	.2488
5	.0295	.0604

Table 2. Values of $\hat{P}(n)$ and $\hat{\hat{P}}(n)$ corresponding to minimum error in Example 8.

IV. Conclusions and Recommendations

A. Conclusions

This report has presented a discussion of the way in which alarms will occur if several radiowave acquisition receiving systems with different intercept probabilities are used to intercept a radio signal. It also gives a least squares method for estimating the intercept probabilities from system performance data. The receiving systems may be collocated or dispersed as in a net.

To use the model described here it is necessary that for each trial the receiving systems alarm in a statistically independent manner. Although only data is required to employ the method described here, hardware may enter into consideration when one attempts to establish independence. Factors such as the method of searching the frequency spectrum, the antenna configuration, propagation effects, the method of signal sorting and validation etc. will all play a role. No claims as to the applicability of the model are made here. Each reader must decide if it may be applied in any particular situation.

A very interesting result that appears here is that given data on the number of signals intercepted one may estimate not only the intercept probabilities but the number of signals which were missed. These are signals which presumably could have been intercepted but were not.

B. Recommendations

The work reported here is certainly not viewed as the final or even the best solution to the problem. The study was undertaken because no

other information on this subject could be found. It presents a solution to the problem which should be of value in the absence of any other. It is hoped that this work may stimulate others to give some thought to this interesting subject.

There are various things which could be pursued in the future.

1. The problem could be cast more elegantly in the language of statistics.

2. The properties and quality of the estimators presented here could be studied.

3. The consequences of receiver dependencies could be examined and perhaps methods developed for handling this.

4. The optimal estimator could be identified.

References

[1] J. A. Boyd et al, Electronics Countermeasures. Ann Arbor, Mich: University of Michigan, 1961.

Appendix I. An Algorithm for Calculating Probability of n Simultaneous Alarms

A. The Algorithm

As discussed in the text, we may calculate the probability of n simultaneous alarms if the intercept probabilities of the receiving systems are known. Let us define

$$\bar{p}_{Ij} = (1-p_{Ij}) \tag{I-1}$$

Then we have

N = 2:

$$P(0) = \bar{p}_{I1}\,\bar{p}_{I2} \tag{I-2a}$$

$$P(1) = \left(\frac{p_{I1}}{\bar{p}_{I1}} + \frac{p_{I2}}{\bar{p}_{I2}}\right) \bar{p}_{I1}\,\bar{p}_{I2} \tag{I-2b}$$

$$P(2) = p_{I1}\,p_{I2} \tag{I-2c}$$

N = 3:

$$P(0) = \prod_{j-1}^{3} \bar{p}_{Ij} \tag{I-3a}$$

$$P(1) = \left(\frac{p_{I1}}{\bar{p}_{I1}} + \frac{p_{I2}}{\bar{p}_{I2}} + \frac{p_{I3}}{\bar{p}_{I3}}\right) \prod_{j-1}^{3} \bar{p}_{Ij} \tag{I-3b}$$

$$P(2) = \left(\frac{\bar{p}_{I1}}{p_{I1}} + \frac{\bar{p}_{I2}}{p_{I2}} + \frac{\bar{p}_{I3}}{p_{I3}}\right) \prod_{j=1}^{3} p_{Ij} \tag{I-3c}$$

$$P(3) = \prod_{j=1}^{3} p_{Ij} \,. \tag{I-3d}$$

Now suppose we know the $P(n)$ and we wish to solve for the p_{Ij}. Define

$$x_j = \frac{p_{Ij}}{\overline{p}_{Ij}} \quad . \tag{I-4}$$

Now by algebraic substitution among Eqs (I-2) or (I-3) we arrive at

N = 2:

$$P(0)\ x_j^2 - P(1)\ x_j + P(2) = 0 \tag{I-5}$$

N = 3:

$$P(0)\ x_j^3 - P(1)\ x_j^2 + P(2)\ x_j - P(3) = 0 \ . \tag{I-6}$$

In general,

$$P(0)\ x_j^N - P(1)\ x_j^{N-1} + P(2)\ x_j^{N-2} - \ldots\ (-1)^N P(N) = 0 \ . \tag{I-7}$$

Thus the x_j are the roots of the polynomial (I-7) and from these roots we may calculate the p_{Ij} from Eq (I-4) as

$$p_{Ij} = \frac{x_j}{1+x_j} \tag{I-8}$$

It is now clear that if the p_{Ij} are known we may obtain a polynomial having the form of Eq (I-7) as

$$\prod_{j=1}^{N} (x-x_j) = a_0 x^N = a_1 x^{N-1} + a_2^{N-2} + \ldots + a_N \ . \tag{I-9}$$

The coefficients, a_n, of this polynomial are proportional to the $P(n)$. Knowing that

$$\sum_{n=0}^{N} P(n) = 1$$

we recognize that

$$P(n) = \frac{|a_n|}{\sum_{k=0}^{N} |a_k|} \quad . \qquad \text{(I-10)}$$

This is the algorithm of Table 1.

B. Discussion of the Algorithm and its Relation to the Least Squares Method

Suppose we had data from which it were possible to estimate the $P(n)$. Then an examination of Eq (I-7) suggests that a possible way of estimating the p_{Ij} is

$$\hat{p}_{Ij} = \text{Re}\left(\frac{x_j}{1+x_j}\right) \qquad \text{(I-11)}$$

where the x_j are the roots of

$$\hat{P}(0)\, x_j^N - \hat{P}(1)\, x_j^{N-1} + \hat{P}(2)\, x_j^{N-2} - \ldots + (-1)^N \hat{P}(N) = 0 \qquad \text{(I-12)}$$

The roots of Eq (I-7) are real. However, if the $P(n)$ are estimated using data and the estimates are used as in Eq (I-12) then some roots may occur in conjugate pairs with small imaginary parts due to the inaccuracy inherent in estimating the $P(n)$ from data. This is why the Re appears in Eq (I-11). Now if the $\hat{p}_{Ij}$ from (I-11) were used in the algorithm above to obtain estimates, $\hat{\hat{P}}(n)$, of the $P(n)$ these estimates would differ from the coefficients $\hat{P}(n)$ in Eq (I-12) because the imaginary part of each root x_j has been discarded. There would thus be some

mean square error

$$E^2 = \sum_{j=0}^{N} \left[\hat{P}(j) - \hat{\hat{P}}(j)\right]^2 \left[\frac{1}{N+1}\right] \qquad \text{(I-13)}$$

which in general would be non-zero.

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